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Higgs Mechanism without Higgs Particle

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Until now there has been no empirical evidence for the existence of the Higgs particle, although the Higgs mechanism of symmetry breaking is very successful. We propose a scalar–tensor theory of gravity with the Higgs field of the $SU(3) \times SU(2) \times U(1)$ standard model of the elementary particles as scalar field, which results finally in Einstein's gravity and in the $SU(3) \times SU(2) \times U(1)$ standard model without any influence of the excited Higgs field.

1. INTRODUCTION

There exists an old idea of Einstein (1913), the so-called "Mach principle of relativity of inertia," according to which mass should be produced by the interaction with the gravitational field. Einstein argued that the inertial mass is only a measure for the resistance of a particle against the relative acceleration with respect to other particles; therefore, within a consequent theory of relativity the mass of a particle should not be given absolutely, but should originate from the interaction with all other particles of the universe, whereby this interaction should be the gravitational one which couples to all particles, i.e., to their masses or energies. He postulated even that the value of the mass of a particle should go to zero if one puts the particle at an infinite distance from all others.

This fascinating idea was not very successful in Einstein's theory of gravity, i.e., general relativity, but it led Brans and Dicke (1961) to develop their scalar–tensor theory of gravity with the intention that the active as well as passive gravitational mass, i.e., the gravitational "constant," be a scalar function determined by the distribution of the particles in the universe.

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[2]The usual partial derivative with respect to the coordinate x^{μ} is denoted by $|\mu$.

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On the other hand, the inertial mass is generated in modern elementary particle physics by the interaction with the scalar Higgs field and we emphasize that the successful Higgs-field mechanism also lies exactly in the direction of Einstein's idea of producing mass by a gravitation-like interaction. One can show (Dehnen *et al.*, 1990; Dehnen and Frommert, 1991) that the Higgs field as source of the inertial mass of the elementary particles mediates a scalar gravitational interaction, however, of Yukawa type, between those particles, which become massive in consequence of the spontaneous symmetry breaking: The masses are the source of the scalar Higgs field and the Higgs field acts back by its gradient on the masses in the momentum law.

Because of the identity of gravitational and inertial mass (equivalence principle), it seems meaningful, if not even necessary, to identify both approaches. For this reason we recently proposed a new scalar–tensor theory of gravity (Dehnen *et al.*, 1992) where the isospin-valued Higgs field of elementary particle physics plays simultaneously the role of a variable gravitational constant instead of the scalar field introduced by Brans and Dicke. The associated Lagrangian which unifies gravity and the other known interactions with a minimum of effort takes the very simple form ($\hbar = 1$, $c = 1$)

$$\mathscr{L} = \left\{\frac{1}{16\pi}\,\alpha\phi^\dagger\phi R + \frac{1}{2}\,\phi^\dagger_{||\mu}\phi^{||\mu} - V(\phi) + L_M\right\}(-g)^{1/2} \tag{1.1}$$

with the Higgs potential

$$V(\phi) = \frac{\mu^2}{2}\,\phi^\dagger\phi + \frac{\lambda}{4!}\,(\phi^\dagger\phi)^2 + \frac{3}{2}\frac{\mu^4}{\lambda} \tag{1.2}$$

the ground-state value of which is normalized to zero, otherwise a cosmological constant would appear; but the Higgs potential will produce a cosmological function. R is the Ricci scalar and α a dimensionless factor which must be determined empirically. The symbol $||\mu$ means the covariant derivative with respect to all external and internal gauge groups.[2] The general form of the Lagrangian (1.1), however, without specifying L_M and the covariant derivative, was proposed already by Zee (1979).

In contrast to our previous paper (Dehnen *et al.*, 1992), where ϕ is considered to be an arbitrary $U(N)$ isovector not contained in the matter Lagrange density L_M [i.e., ϕ does not produce the fermionic masses like, for example, the $SU(5)$ GUT Higgs field], we restrict ourselves here to the minimal standard model of the internal gauge group $SU(3)\times SU(2)\times U(1)$ with the $SU(2)\times U(1)$ Higgs field ϕ generating all the fermionic and the

$W^{\pm}$ and Z^0 masses. In this case the matter Lagrangian reads explicitly (L, R mean summation of left-, right-handed terms)

$$L_M = \frac{i}{2}\bar{\psi}\gamma^{\mu}_{L,R}\psi_{||\mu} + \text{h.c.} - \frac{1}{16\pi}F^a_{\mu\nu}F^{\mu\nu}_a - k\bar{\psi}_R\phi^{\dagger}\hat{x}\psi_L + \text{h.c.} \qquad (1.3)$$

Herein ψ summarizes the leptonic and hadronic Dirac wave-functions, $F_{\mu\nu a}$ are the gauge-field strengths, and $\hat{x}$ represents the Yukawa coupling matrix for the fermionic masses. With exception of μ^2, the Lagrangian (1.1) together with (1.2) and (1.3) does not contain any dimensional parameter (λ and k are dimensionless real-valued constants).

The essential result of such a theory turns out to be that after spontaneous symmetry breaking the excited Higgs field possesses no sources. As a consequence the Higgs field cannot be excited by other elementary particles, so that no Higgs particles can be generated in high-energy experiments.

2. FIELD EQUATIONS

The field equations following from the action principle associated with (1.1) are generalized Einstein equations

$$\begin{aligned} R_{\mu\nu} &- \frac{1}{2}Rg_{\mu\nu} + \frac{8\pi}{\alpha\phi^{\dagger}\phi}V(\phi)\,g_{\mu\nu} \\ &= -\frac{8\pi}{\alpha\phi^{\dagger}\phi}T_{\mu\nu} - \frac{8\pi}{\alpha\phi^{\dagger}\phi}\left[\phi^{\dagger}_{(||\mu}\phi_{||\nu)} - \frac{1}{2}\phi^{\dagger}_{||\lambda}\phi^{||\lambda}g_{\mu\nu}\right] \\ &\quad - \frac{1}{\phi^{\dagger}\phi}\left[(\phi^{\dagger}\phi)_{|\mu||\nu} - (\phi^{\dagger}\phi)^{|\lambda}{}_{||\lambda}\,g_{\mu\nu}\right] \end{aligned} \qquad (2.1)$$

the Higgs field equation

$$\phi^{||\mu}{}_{||\mu} - \frac{\alpha}{8\pi}R\phi + \left(\mu^2 + \frac{\lambda}{6}\phi^{\dagger}\phi\right)\phi = -2k\bar{\psi}_R\hat{x}\psi_L \qquad (2.2)$$

the Dirac equations

$$i\gamma^{\mu}_{\binom{L}{R}}\psi_{||\mu} - k\begin{pmatrix}\hat{x}^{\dagger}\phi\psi_R \\ \phi^{\dagger}\hat{x}\psi_L\end{pmatrix} = 0 \qquad (2.3)$$

and the inhomogeneous Yang–Mills equations for the gauge field strengths

$$F_a^{\nu\mu}{}_{||\nu} = 4\pi j^{\mu}_a \qquad (2.4)$$

with the gauge currents

$$j_a^\mu = j_a^\mu(\psi) + j_a^\mu(\phi) = g\bar{\psi}\gamma^\mu_{L,R}\tau_a\psi + \frac{ig}{2}\phi^\dagger\tau_a\phi^{\|\mu} + \text{h.c.} \tag{2.4a}$$

belonging to the fermions and the Higgs field, respectively. In (2.4a) τ_a are the generators of the internal unitary gauge group in question and g means the corresponding gauge coupling constants.

The energy-momentum tensor $T_{\mu\nu}$ in (2.1) is the symmetric metrical one belonging to (1.3) and reads, with the use of the Dirac equation (2.3),

$$T^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^{(\mu}_{L,R}\psi^{\|\nu)} + \text{h.c.} - \frac{1}{4\pi}\left(F^{\mu a}{}_{\lambda}F_a^{\nu\lambda} - \frac{1}{4}F^a_{\alpha\beta}F_a^{\alpha\beta}g^{\mu\nu}\right) \tag{2.5}$$

In consequence of the coupling with the Higgs field the energy-momentum law is modified in comparison with the pure Einstein theory; one has

$$T_\mu{}^\nu{}_{\|\nu} = k\bar{\psi}_R\phi^\dagger_{\|\mu}\hat{x}\psi_L + \text{h.c.} + F^a_{\nu\mu}j_a^\nu(\phi) \tag{2.5a}$$

Finally we note that the purely gravitational part of the theory is related to a generalization of Brans and Dicke's theory proposed by Bergmann (1968) and Wagoner (1970).

3. SPONTANEOUS SYMMETRY BREAKING

From the field equation (2.2) it follows for the nontrivial Higgs-field ground state ($\mu^2 < 0$)

$$\phi_0 = vN, \qquad N^\dagger N = 1, \qquad N = \text{const}, \qquad v^2 = \phi_0^\dagger\phi_0 = -\frac{6\mu^2}{\lambda} \tag{3.1}$$

for which the Higgs potential (1.2) is minimized simultaneously; its ground-state value is zero and herewith the field equations (2.1) for the metric are fulfilled identically with the Minkowski metric as the metrical ground state. Insertion of (3.1) into the Dirac equation (2.3) and into the Higgs-field gauge current (2.4a) of the Yang–Mills equation (2.4) yields the fermionic mass matrix

$$\hat{m} = \tfrac{1}{2}kv(N^\dagger\hat{x} + \hat{x}^\dagger N) \tag{3.2}$$

and the matrix of the mass square of the gauge bosons:

$$M^2_{ab} = 4\pi g^2v^2N^\dagger\tau_{(a}\tau_{b)}N \tag{3.3}$$

The diagonalization procedure of (3.3) is not given here explicitly but results in the usual Weinberg mixing with the bosonic masses $M_W = \pi^{1/2} g_2 v$, $M_Z = v[\pi(g_1^2 + g_2^2)]^{1/2}$, and $M_A = 0$ (the gluons remain massless of course).

In the unitary gauge the Higgs field ϕ takes the form, avoiding Goldstone bosons,

$$\phi = \rho N, \qquad \rho^2 = \phi^\dagger \phi \tag{3.4}$$

Referring ρ to the ground state, we set

$$\rho = v(1+\varphi) \tag{3.4a}$$

where the real-valued scalar field φ describes the excited Higgs field connected with the Higgs particle [see equation (3.8)]. Insertion of (3.4) and (3.4a) into (2.1) gives the field equation of gravitation:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} &+ \frac{12\pi}{\alpha v^2} \frac{\mu^4}{\lambda} \frac{[(1+\varphi)^2 - 1]^2}{(1+\varphi)^2} g_{\mu\nu} \\ &= -\frac{8\pi}{\alpha v^2 (1+\varphi)^2} \left[T_{\mu\nu} + \frac{1}{4\pi} (1+\varphi)^2 M_{ab}^2 \left(A_\mu^a A_\nu^b - \frac{1}{2} A_\lambda^a A^{b\lambda} g_{\mu\nu} \right) \right. \\ &\quad \left. + v^2 \left(\varphi_{|\mu} \varphi_{|\nu} - \frac{1}{2} \varphi_{|\lambda} \varphi^{|\lambda} g_{\mu\nu} \right) \right] \\ &\quad - \frac{1}{(1+\varphi)^2} [(1+\varphi)^2_{|\mu||\nu} - (1+\varphi)^{2|\lambda}{}_{||\lambda} g_{\mu\nu}] \end{aligned} \tag{3.5}$$

with the trace

$$\begin{aligned} R = \frac{8\pi}{\alpha v^2 (1+\varphi)^2} &\left\{ T - \frac{1}{4\pi} (1+\varphi)^2 M_{ab}^2 A_\lambda^a A^{b\lambda} \right. \\ &\left. - v^2 \varphi_{|\lambda} \varphi^{|\lambda} + 6 \frac{\mu^4}{\lambda} [(1+\varphi)^2 - 1]^2 \right\} - \frac{3}{(1+\varphi)^2} (1+\varphi)^{2|\lambda}{}_{||\lambda} \end{aligned} \tag{3.5a}$$

and the equation of motion [see (2.5a)]

$$\begin{aligned} &\left[T_\mu{}^\nu + \frac{(1+\varphi)^2}{4\pi} M_{ab}^2 \left(A_\mu^a A^{b\nu} - \frac{1}{2} \delta_\mu{}^\nu A_\lambda^a A^{b\lambda} \right) \right]_{||\nu} \\ &\qquad = \varphi_{|\mu} \left[\bar{\psi} \hat{m} \psi - \frac{1+\varphi}{4\pi} M_{ab}^2 A_\lambda^a A^{b\lambda} \right] \end{aligned} \tag{3.5b}$$

($A_{\mu a}$ are the gauge potentials). According to (3.5b), the Higgs field φ generates a gravitation-like potential force acting on the massive particles (Dehnen *et al.*, 1990; Dehnen and Frommert, 1991).

Obviously the Newtonian gravitational constant is defined only after symmetry breaking (Adler, 1982) and is given by [see (3.5)]

$$G = 1/\alpha v^2 \tag{3.6}$$

whereas its variability is described by $(1+\varphi)^{-2}$. Simultaneously equation (3.6) determines the value of the parameter α because v^2 is known from the masses (3.3) [see also (3.9)]. The cosmological function originated by the Higgs potential is necessarily positive and possesses the value

$$\Lambda = 12\pi G \frac{\mu^4}{\lambda} \frac{[(1+\varphi)^2 - 1]^2}{(1+\varphi)^2} \tag{3.7}$$

It vanishes for the ground state ($\varphi = 0$). The first bracket on the right-hand side of (3.5) represents the effective energy-momentum tensor of fermions and gauge bosons taking additionally into account the masses of the gauge bosons and energy and momentum of the excited Higgs field.

In the same way it follows from (2.2) after insertion of (3.5a) and (3.6) that

$$[(1+\varphi)^2 - 1]^{|\mu}{}_{||\mu} + M^2[(1+\varphi)^2 - 1] = \frac{1}{1 + 4\pi/3\alpha} \frac{8\pi G}{3} [T - (1+\varphi)\bar{\psi}\hat{m}\psi] \tag{3.8}$$

with the square of the Higgs mass

$$M^2 = \frac{16\pi G(\mu^4/\lambda)}{(1 + 4\pi/3\alpha)} \tag{3.8a}$$

The comparison of (3.3) and (3.6) gives the value of α; one finds immediately

$$\alpha \simeq (M_{Pl}/M_W)^2 \simeq 10^{33} \tag{3.9}$$

where $M_{Pl} = 1/\sqrt{G}$ is the Planck mass. Accordingly, the value of (3.8a) is smaller than the usual one by the small factor α^{-1} and of the same order of magnitude as Λ; see (3.7). In consequence of the nonminimal coupling of gravitational and Higgs fields in (1.1) and (2.2), the trace T appears as an additional source in (3.8), which results later in a total cancellation of the right-hand side.

Finally we give the Dirac equation (2.3) and the Yang–Mills equations (2.4) after symmetry breaking. One obtains with (3.2)

$$i\gamma^{\mu}_{\binom{L}{R}}\psi_{\|\mu} - (1+\varphi)\,\hat{m}\psi_{\binom{R}{L}} = 0 \tag{3.10}$$

and with (3.3)

$$F_{a}^{\nu\mu}{}_{\|\nu} + (1+\varphi)^2\, M^2_{ab} A^{\mu b} = 4\pi j^{\mu}_{a}(\psi) \tag{3.11}$$

Now we are able to calculate the source of the excited Higgs field φ according to (3.8). From (2.5) and (3.10) we get for the trace of $T^{\mu\nu}$

$$T = \frac{i}{2}\,\bar{\psi}\gamma^{\mu}_{L,R}\psi_{\|\mu} + \text{h.c.} = (1+\varphi)\,\bar{\psi}\hat{m}\psi \tag{3.12}$$

Evidently by insertion of (3.12) into (3.18) the source of the excited Higgs field φ vanishes identically; it obeys exactly the homogeneous (nonlinear) wave equation:

$$\varphi^{|\mu}{}_{\|\mu} + M^2\varphi\,\frac{(1+\varphi/2)}{1+\varphi} + \frac{\varphi_{|\mu}\varphi^{|\mu}}{1+\varphi} \equiv 0 \tag{3.13}$$

or in view of (3.8) the homogeneous Klein–Gordon equation for the variable $(1+\varphi)^2 - 1$.

We note here explicitly that not only the fermionic masses, but also those of the gauge bosons no longer appear in (3.13) as source for the excited Higgs field; it is coupled only to the very weak gravitational field contained in the only space-time covariant derivative. In the case of several Higgs fields the cancellation discussed above takes place only in the field equation for that particular Higgs field generating the gravitational constant and is true only for those masses produced by that particular Higgs field.

4. CONCLUSIONS

Obviously, in consequence of equation (3.13) it is practically impossible to generate the Higgs field φ or the associated particle of mass M in the laboratory; in view of the space-time covariant derivative this would be possible only in the extremely weak gravitational channel. Otherwise the φ field can exist as a cosmological background field and perhaps solve the dark matter problem. In any case the solution $\varphi \equiv 0$ is always possible. Herewith we obtain from (3.5) and (3.5b) the usual Einstein gravity (without cosmological constant) with fermionic and bosonic energy-momentum tensor as source, and from (3.10) and (3.11) the Dirac equations, and the Yang–Mills equations for the $SU(3)\times SU(2)\times U(1)$ standard model follow without any influence of the excited Higgs field φ. Instead of this there exists only the very weak gravitational interaction.

Of course, the foregoing calculations are performed classically. However, a quantization is possible in a straightforward manner, if we restrict ourselves with respect to the gravitational field to its linearized weak-field version, which is completely sufficient for the low-energy limit discussed above.

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